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A Method for the Rapidly Convergent Representation of Electromagnetic Fields in a Rectangular Waveguide

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Abstract—**In numerically analyzing the electromagnetic fields in a rectangular waveguide by the integral equation method, it is essential to calculate the electromagnetic fields produced by the electric and magnetic currents and charges. In this paper, a new method is proposed to facilitate rapid calculation of the three-dimensional fields. This method is a modified image method and gives an accurate value in a short time.**

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I. Introduction

The integral equation method has been effectively used to analyze electromagnetic fields in a rectangular waveguide [1]–[3]. In solving integral equations, it is desirable to calculate the fields produced by electromagnetic sources, such as electric and magnetic currents and charges, within a short time.

The fields have usually been expressed by the mode method or the image method [4]. In the mode method, the field is expressed by a series using modal functions which satisfy the boundary conditions on the waveguide wall (here called the mode series). In the image method, the field is expressed by a series which is composed of fields produced by a source and its images arranged to infinity in order to satisfy the boundary conditions on the waveguide wall (here called the image series). As the fields are expressed by infinite series, the convergence of the series is sometimes troublesome and several methods have been proposed for rapid computation. There is a rapid convergence method [1], [2] for two-dimensional fields and a modified image method [3] for three-dimensional fields.

A modified image method for calculating the fields produced by straight electric and magnetic current segments has been proposed and found to be effective for calculating the fields near the source. Computing errors of the fields increase as the distance from the source to the observation point increases. Therefore, the modified image method is improved for the accurate calculation at any position by adding a correcting term. In order to check the adequacy and usefulness of the method considering the term, its error is compared with that of the method without it.

II. Representations of the Electromagnetic Fields

The electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{H}$, in the Lorentz gauge are expressed by using the vector potentials $\boldsymbol{A}$ and $\boldsymbol{A}^*$ produced respectively by electric and magnetic currents $\boldsymbol{i}$ and $\boldsymbol{i}^*$ and the scalar potentials ϕ and ϕ^* produced respectively by electric and magnetic charges ρ and ρ^* as follows [5]:

$$\boldsymbol{E} = -j\omega \boldsymbol{A} - \nabla\phi - \nabla \times \boldsymbol{A}^* / \epsilon_0 \tag{1}$$

$$\boldsymbol{H} = -j\omega \boldsymbol{A}^* - \nabla\phi^* + \nabla \times \boldsymbol{A} / \mu_0 \tag{2}$$

where ϵ_0 and μ_0 are, respectively, the permittivity and the permeability and $\omega = 2\pi f$ with a frequency f.

Setting the Cartesian coordinates with the z axis parallel to the direction of wave propagation and the x and y axes parallel to each wall of the waveguide, we obtain the vector and scalar potentials at $P_0(x_0, y_0, z_0)$ produced by the currents and charges positioned at $P_s(x_s, y_s, z_s)$.

A. Representation of the Fields by the Image Method

The potentials are given equivalently by adding those produced by electromagnetic sources such as $\boldsymbol{i}$, $\boldsymbol{i}^*$, ρ, and ρ^* and their images $\boldsymbol{i}_t$, $\boldsymbol{i}_t^*$, ρ_t, and ρ_t^* [3].

The images are positioned at $P_t(x_t, y_t, z_t)$:

$$x_t = 2am + (-1)^k x_s \qquad y_t = 2bn + (-1)^l y_s \qquad z_t = z_s$$

with a being the width and b the height of the waveguide and $k = 0$ or 1, $l = 0$ or 1, $m = \cdots, -3, -2, -1, 0, 1, 2, 3, \cdots$, and $n = \cdots, -3, -2, -1, 0, 1, 2, 3, \cdots$.

The direction of $\boldsymbol{i}_t$ is the same as that of $\boldsymbol{i}$ in the case where $k = l$ and the opposite in the case where $k \neq l$. The direction of $\boldsymbol{i}_t^*$ is the same as that of $\boldsymbol{i}^*$ at any position. The polarity of ρ_t is the same as that of ρ in the case of $k = l$ and the opposite in the case of $k \neq l$. The polarity of ρ_t^* is the same as that of ρ^* at any position.

The potentials are given as follows:

$$\boldsymbol{A} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{k=0}^{1} \sum_{l=0}^{1} \frac{\mu_0 \exp(-jk_0 r_t)}{4\pi r_t} \boldsymbol{i}_t \quad (3)$$

$$\boldsymbol{A}^* = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{k=0}^{1} \sum_{l=0}^{1} \frac{\epsilon_0 \exp(-jk_0 r_t)}{4\pi r_t} \boldsymbol{i}_t^* \quad (4)$$

$$\phi = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{k=0}^{1} \sum_{l=0}^{1} \frac{\exp(-jk_0 r_i)}{4\pi \epsilon_0 r_t} \rho_t \quad (5)$$

$$\phi^* = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{k=0}^{1} \sum_{l=0}^{1} \frac{\exp(-jk_0 r_t)}{4\pi \mu_0 r_t} \rho_t^* \quad (6)$$

where

$$k_0 = j\omega\sqrt{\epsilon_0 \mu_0} \text{ and } r_t = \sqrt{(x_0 - x_t)^2 + (y_0 - y_t)^2 + (z_0 - z_t)^2}.$$

B. Representation of the fields by the Mode Method

The potentials are given as follows [6]:

$$\boldsymbol{A} = \mu_0 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} E_{mn}\big(C_{m0}C_{ms}S_{n0}S_{ns}i_x \boldsymbol{x} + S_{m0}S_{ms}C_{n0}C_{ns}i_y \boldsymbol{y} + S_{m0}S_{ms}S_{n0}S_{ns}i_z \boldsymbol{z}\big) \quad (7)$$

$$\boldsymbol{A}^* = \epsilon_0 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} E_{mn}\big(S_{m0}S_{ms}C_{n0}C_{ns}i_x^* \boldsymbol{x} + C_{m0}C_{ms}S_{n0}S_{ns}i_y^* \boldsymbol{y} + C_{m0}C_{ms}C_{n0}C_{ns}i_z^* \boldsymbol{z}\big) \quad (8)$$

$$\phi = \frac{1}{\epsilon_0} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} E_{mn} S_{m0} S_{ms} S_{n0} S_{ns} \rho \quad (9)$$

$$\phi^* = \frac{1}{\mu_0} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} E_{mn} C_{m0} C_{ms} C_{n0} C_{ns} \rho^* \quad (10)$$

where the subscripts x, y, and z on i and i^* denote the x, y, and z components, respectively; $\boldsymbol{x}, \boldsymbol{y}$, and $\boldsymbol{z}$ denote the unit vectors in the x, y, and z directions; and E_{mn} stands for

$$E_{mn} = \frac{(2-\delta_m)(2-\delta_n)}{2abk_{mn}} \exp(-k_{mn}|z_0 - z_s|). \quad (11)$$

When $m, n = 0$, δ_m, $\delta_n = 1$; otherwise, δ_m, $\delta_n = 0$. Also,

$$k_{mn} = \sqrt{k_m^2 + k_n^2 - k_0^2} \qquad k_m = m\pi/a \qquad k_n = n\pi/b$$

$$S_{m0} = \sin(k_m x_0) \qquad C_{m0} = \cos(k_m x_0)$$

$$S_{n0} = \sin(k_n y_0) \qquad C_{n0} = \cos(k_n y_0)$$

$$S_{ms} = \sin(k_m x_s) \qquad C_{ms} = \cos(k_m x_s)$$

$$S_{ns} = \sin(k_n y_s) \qquad C_{ns} = \cos(k_n y_s).$$

C. A Rapidly convergent Representation of the Fields

As every term of the mode series contains an exponential function as given in (11), the convergence of the series is rapid even though the differential operations of the second and third terms in (1) and (2) make it more difficult. However, as $|z_0 - z_s|$ approaches zero, it rapidly becomes more difficult. In order to facilitate a rapid calculation even in this case, a new method is proposed which advantageously uses the mode and image methods.

The field $\boldsymbol{F}$ obtained by the image method is divided into two parts: a near field produced by a source and its images near the source (here called the near images) and a far field produced by the images far from the source extending to infinity.

Taking the near images from $-M$ to M in the x direction and from $-N$ to N in the y direction and adding the fields $\boldsymbol{F}_i$ produced by them, we get the near field $\boldsymbol{F}_{ig1}$ as

$$\boldsymbol{F}_{ig1} = \sum_{m=-M}^{M} \sum_{n=-N}^{N} \sum_{k=0}^{1} \sum_{l=0}^{1} \boldsymbol{F}_t. \quad (12)$$

The sum forming the far field $\boldsymbol{F}_{ig2}$ can be obtained with the help of the sum of the mode series $\boldsymbol{F}_{md}$ as

$$\boldsymbol{F}_{ig2} = \boldsymbol{F}_{md} - \boldsymbol{F}_{ig1}. \quad (13)$$

As the convergence of the mode series is difficult when $|z_0 - z_s|$ is small, let a temporary source be set at $P_t(x_s, y_s, z_t)$ at some distance from P_s, and let the far field be expressed as

$$\boldsymbol{F}_{ig3} = \boldsymbol{F}_{ig2}|_t + \delta \boldsymbol{F}_{ig} \quad (14)$$

where the symbol $|_t$ denotes the value given by the temporary source and $\delta \boldsymbol{F}_{ig}$ is the correcting term. Then, the field $\boldsymbol{F}$ is expressed as

$$\boldsymbol{F} = \boldsymbol{F}_{ig1} + \boldsymbol{F}_{ig3}. \quad (15)$$

The correcting term is given in the following way. The terms of the image series of the fields obtained by substituting (5)–(8) into (1) and (2) contain two types of functions of z as

$$F_1 = C_1 \frac{\exp(-jk_0 r_t)}{r_t^L} \quad (16)$$

$$F_2 = C_2(z_0 - z) \frac{\exp(-jk_0 r_i)}{r_t^L} \quad (17)$$

where C_1 and C_2 are constants or functions of x or y, and $L = 1$ for the first term field and $L = 2$ or 3 for the second and third term fields in (1) and (2).

Differentiating F_1 and F_2 of the far fields with respect to z and considering $r_t \gg |z_0 - z|$, we derive respectively

$$\frac{\partial F_{ig2}}{\partial z} = \frac{\partial F_{ig2}}{\partial z}\bigg|_t \frac{z_0 - z}{z_0 - z_t} \quad (18)$$

$$\frac{\partial F_{ig2}}{\partial z} = \frac{-1}{z_0 - z_t} F_{ig2}|_t + \left(\frac{\partial F_{ig2}}{\partial z}\bigg|_t + \frac{1}{z_0 - z_t} F_{ig2}|_t \right) \frac{(z_0 - z)^2}{(z_0 - z_t)^2} \quad (19)$$

where F_{ig2} denotes the x, y, or z component of $\boldsymbol{F}_{ig2}$.

Integrating (18) and (19) from z_t to z_s, any of the correcting terms $\delta \boldsymbol{F}_{ig}$ can be obtained.

III. Examination of the New Field Calculation

Using a rectangular waveguide, R26 ($a = 8.6$ cm, $b = 4.3$ cm), with infinitesimally small sources such as $\boldsymbol{i}$, $\boldsymbol{i}^*$, ρ, and ρ^* of unit amplitude ($f = 2.45$ GHz), the errors of the fields computed by the new method are examined. The currents and charges are

TABLE I
MAXIMUM ERRORS, ϵ_{max}, [%] OF THE FIELDS CALCULATED BY THE NEW METHOD

	E_{ax}	E_{ay}	E_{az}	E_c	E_{bx}	E_{by}	E_{bz}
Field [V/cm]	0.9897 $\times 10^{-2}$	0.3505	0.5971 $\times 10^{-3}$	0.1816 $\times 10^{-1}$	0.1042 $\times 10^{-3}$	0.7318 $\times 10^{-5}$	0.2191 $\times 10^{-2}$
ϵ_{max}	−0.038	0.0007	−0.065	−0.0037	0.0045	0.111	0.0006
P_s	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(5,1)
P_o	(10,10)	(1,10)	(10,1)	(10,2)	(10,10)	(10,10)	(10,10)
N_{con}	4057	2936	4057	4057	4057	4057	3472
F_{con}	1×10^{-11}	1×10^{-9}	1×10^{-11}	1×10^{-11}	1×10^{-11}	1×10^{-11}	1×10^{-10}
	H_{ax}	H_{ay}	H_{az}	H_c	H_{bx}	H_{by}	H_{bz}
Field [A/cm]	0.247 $\times 10^{-5}$	0.6975 $\times 10^{-7}$	0.3472 $\times 10^{-5}$	0.1688 $\times 10^{-4}$	0.1525 $\times 10^{-3}$	0.5685 $\times 10^{-2}$	0.248 $\times 10^{-4}$
ϵ_{max}	0.0007	−0.038	−0.0016	−0.0022	0.035	−0.0006	−0.0039
P_s	(1,1)	(1,1)	(3,2)	(5,1)	(4,1)	(1,5)	(1,1)
P_o	(1,10)	(10,10)	(7,1)	(10,10)	(8,5)	(2,1)	(10,2)
N_{con}	2936	4057	2936	3472	2936	1988	4057
F_{con}	1×10^{-9}	1×10^{-11}	1×10^{-9}	1×10^{-10}	1×10^{-9}	1×10^{-7}	1×10^{-11}

TABLE II
MAXIMUM ERRORS, ϵ_{max}, [%] OF THE FIELDS CALCULATED WITHOUT THE CORRECTING TERMS

	E_{ax}	E_{ay}	E_{az}	E_c	E_{bx}	E_{by}	E_{bz}
Field [V/cm]	0.9941 $\times 10^{-2}$	0.3778	0.5842 $\times 10^{-3}$	0.1171 $\times 10^{-1}$	0.1042 $\times 10^{-3}$	0.7492 $\times 10^{-5}$	0.1963 $\times 10^{-2}$
ϵ_{max}	−62.7	−0.75	38.2	0.84	−19.8	610	−0.45
P_s	(1,1)	(1,2)	(1,1)	(1,1)	(1,1)	(1,1)	(4,5)
P_o	(10,1)	(1,9)	(10,10)	(10,1)	(10,10)	(10,1)	(10,9)
N_{con}	2443	1988	2936	2443	2936	2443	1988
F_{con}	1×10^{-8}	1×10^{-7}	1×10^{-9}	1×10^{-8}	1×10^{-9}	1×10^{-8}	1×10^{-7}
	H_{ax}	H_{ay}	H_{az}	H_c	H_{bx}	H_{by}	H_{bz}
Field [A/cm]	0.2663 $\times 10^{-5}$	0.7003 $\times 10^{-7}$	0.2896 $\times 10^{-5}$	0.1688 $\times 10^{-4}$	0.3812 $\times 10^{-4}$	0.6579 $\times 10^{-2}$	0.1599 $\times 10^{-4}$
ϵ_{max}	−0.75	−62.7	−1.02	−1.01	71.3	0.68	6.52
P_s	(1,2)	(1,1)	(4,3)	(5,1)	(1,1)	(5,1)	(1,1)
P_o	(1,9)	(10,1)	(8,1)	(10,10)	(10,5)	(6,9)	(10,1)
N_{con}	1988	2443	1988	2443	2443	1988	2443
F_{con}	1×10^{-7}	1×10^{-8}	1×10^{-7}	1×10^{-8}	1×10^{-8}	1×10^{-7}	1×10^{-8}

positioned at $P_s(x_s, y_s, z_s)$ on the plane of $z = 0$:

$$x_s = 0.1a(n_{sx} - 0.5) \qquad y_s = 0.1b(n_{sy} - 0.5) \qquad z_s = 0,$$

$$n_{sx} = 1, 2, \cdots, 5; \qquad n_{sy} = 1, 2, \cdots, 5 \tag{20}$$

and the observation point is positioned at $P_o(x_o, y_o, z_o)$ on the plane of $z = 0.05a$:

$$x_o = 0.1a(n_{0x} - 0.5) \qquad y_o = 0.1b(n_{oy} - 0.5) \qquad z_o = 0.5a,$$

$$n_{ox} = 1, 2, \cdots, 10; \qquad n_{oy} = 1, 2, \cdots, 10. \tag{21}$$

The maximum errors, ϵ_{max}, of the fields computed by the new method are shown in Table I and those without the correcting term in Table II. In these tables, the source and observation points, P_s and P_o, are represented by (n_{sx}, n_{sy}) and (n_{ox}, n_{oy}) in (20) and (21), respectively, in which case the maximum error, ϵ_{max}, occurs:

$$\epsilon_{max} = (|\boldsymbol{T}| - |\boldsymbol{M}|)/|\boldsymbol{T}| \times 100\ [\%]$$

with the field $\boldsymbol{M}$ computed by the new method and the sufficiently convergent value $\boldsymbol{T}$ computed by the mode method. The fields shown in the tables are the values of $\boldsymbol{T}$.

The symbols denoting the fields in these tables represent

$$E_{ax} = -j\omega A_x \qquad E_{ay} = -j\omega A_y \qquad E_{az} = -j\omega A_z$$

$$E_c = -\nabla\phi \qquad E_{bx} = -\nabla \times A_x^*/\epsilon_0 \qquad E_{by} = -\nabla \times A_y^*/\epsilon_0$$

$$E_{bz} = -\nabla \times A_z^*/\epsilon_0$$

$$H_{ax} = -j\omega A_x^* \qquad H_{ay} = -j\omega A_y^* \qquad H_{az} = -j\omega A_z^*$$

$$H_c = -\nabla\phi^* \qquad H_{bx} = \nabla \times A_x/\mu_0$$

$$H_{by} = \nabla \times A_y/\mu_0 \qquad H_{bz} = \nabla \times A_z/\mu_0$$

where the subscripts x, y, and z on $\boldsymbol{A}$ and $\boldsymbol{A}^*$ denote the vector potentials produced by the electric and magnetic currents in the x, y, and z directions, respectively.

In calculating the fields, a temporary source is set by shifting the actual source in the z direction by $0.05a$, and 289 near images (17×17 images in the x and y directions) are considered.

The convergence of the mode series is judged by the convergent condition; that is,

$$|V_n - V_0|/|V_n| < \delta_c$$

where V_0 is a partial sum truncated when $\exp(-k_{mn}|z_o - z_s|)$ reaches F_{con} and V_n is a new partial sum, obtained by decreasing F_{con} to $F_{\text{con}}/10$. In this examination, δ_c is set at 0.001 and an initial F_{con} is set at 0.001.

The number of additions, N_{con}, until the convergence of the mode series is shown in conjunction with the final F_{con}. As the correcting term δF_{tg} converges slowly, N_{con} is large compared with that without the term. However, the term lessens remarkably the computing errors.

There are various choices of what method to use in computing the fields with errors of less than 1% within a given time, and one choice is as follows: If $|z_o - z_s| > 0.1a$, the mode method is effective; if $|z_o - z_s| < 0.1a$, the new method is effective for $|z_t - z_s| = 0.05a$ and 289 near images (17×17 images in the x and y directions); and if the distance from P_s to P_o is less than $0.001a$, the image method is effective, especially for the second and third term fields in (1) and (2) because they are mainly determined by the source and images close to the source.

IV. Conclusions

A new method for rapidly computing the electromagnetic fields in a rectangular waveguide has been proposed. The method is a modified image method utilizing the rapid convergence of the mode series with some space between the source and observation points in the direction of wave propagation.

The modified image method without the correcting terms can be used only for evaluating the field near the source. However, by considering the terms, the method can be effectively used for the field at any position. Even if the new method requires a computation of the fields expressed by both the image and mode methods, the computing time is short because, in the image method, only a few near images are taken into account and, in the mode method, $|z_o - z_t|$ is chosen so large that even the series of the correcting term converges rapidly.

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